Essential Tool for Physics Laws, Concepts, Variables \& Equations, Including \& Sample Problems, $\triangle$ Common Pitfalls \& $\rho$ Helpful Hints

## BAGIG

| A. Units for Physical Quantities |  |  |
| :---: | :---: | :---: |
| Base Units | Symbol | Unit |
| Length | $l, x$ | Meter - m |
| Mass | $m, M$ | Kilogram - kg |
| Temperature | $T$ | Kelvin - K |
| Time | $t$ | Second - s |
| Electric Current | $I$ | Ampere - A (C/s) |
| Derived Units | Symbol | Unit |
| Acceleration | $a$ | $\mathrm{m} / \mathrm{s}^{2}$ |
| Ang. Accel. | $\alpha$ | radian/s ${ }^{2}$ |
| Ang. Momentum | $L$ | $\mathrm{kg} \mathrm{m} / \mathrm{s}$ |
| Ang. Velocity | $\omega$ | radian/sec |
| Angle | $\theta, \varphi$ | radian |
| Capacitance | C | Farad F (C/V) |
| Charge | $Q, q, e$ | Coulomb C (A s) |
| Density | $\rho$ | $\mathrm{kg} / \mathrm{m}^{3}$ |
| Displacement | $s, d, h$ | meter - m |
| Electric Field | E | V/m |
| Electric Flux | $\Phi_{e}$ | V m |
| Electromotive Force (EMF) | E | Volt - V |
| Energy | E, U, K | Joule $\mathrm{J}\left(\mathrm{kg} \mathrm{m}^{2} \mathrm{~s}^{-2}\right)$ |
| Entropy | $S$ | J/K |
| Force | $F$ | Newton - $\mathrm{N}\left(\mathrm{kg} \mathrm{m} / \mathrm{s}^{2}=\mathrm{J} / \mathrm{m}\right)$ |
| Frequency | $f, v$ | Hertz - Hz (cycle/s) |
| Heat | $Q$ | Joule - J |
| Magnetic Field | B | Tesla ( $\mathrm{Wb} / \mathrm{m}^{2}$ ) |
| Magnetic Flux | $\Phi_{m}$ | Weber Wb ( $\mathrm{kg} \mathrm{m}^{2} / \mathrm{As}^{2}$ ) |
| Momentum | $p$ | kg m/s |
| Potential | V | Voltage V (J/C) |
| Power | P, P | Watt - W (J/s) |
| Pressure | $P$ | Pascal - $\mathrm{Pa}\left(\mathrm{N} / \mathrm{m}^{2}\right.$ ) |
| Resistance | $R$ | Ohm $\Omega$ (V/A) |
| Torque | $\tau$ | Nm |
| Velocity | $v$ | $\mathrm{m} / \mathrm{s}$ |
| Volume | V | $\mathrm{m}^{3}$ |
| Wavelength | $\lambda$ | meter - m |
| Work | W | Joule - J ( m ) |

B. Fundamental Physical Constants

| Base Units | Symbol | Unit |
| :--- | :---: | :--- |
| Mass of electron | $m_{\mathrm{e}}$ | $9.11 \times 10^{-31} \mathrm{~kg}$ |
| Mass of proton | $m_{\mathrm{p}}$ | $1.67 \times 10^{-27} \mathrm{~kg}$ |
| Avogadro Constant | $N_{\mathrm{A}}$ | $6.022 \times 10^{23} \mathrm{~mol}^{-1}$ |
| Elementary charge | $e$ | $1.602 \times 10^{-19} \mathrm{C}$ |
| Faraday Constant | $F$ | $96,485 \mathrm{C} / \mathrm{mol}$ |
| Speed of light | $c$ | $3 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$ |
| Molar Gas Constant | $R$ | $8.314 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$ |
| Boltzmann Constant | $k$ | $1.38 \times 10^{-23} \mathrm{~J} \mathrm{~K}^{-1}$ |
| Gravitation Constant | $G$ | $6.67 \times 10^{-11} \mathrm{~m}^{3} \mathrm{~kg}^{-1} \mathrm{~s}^{-2}$ |
| Permeability of Space | $\mu_{0}$ | $4 \pi \times 10^{-7} \mathrm{~N} / \mathrm{A}^{2}$ |
| Permittivity of Space | $\varepsilon_{0}$ | $8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}^{2}$ |

C. Conversion factors and alternative units

|  | Unit | Description |
| :--- | :--- | :--- |
| Angle | ${ }^{\circ}$ (degree) | $180^{\circ}=\pi \mathrm{rad}$ |
| Energy | Erg | CGS unit $\left(\mathrm{g} \mathrm{cm}^{2} / \mathrm{s}^{2}\right)$ <br> $1 \mathrm{erg}=10^{-7} \mathrm{~J}$ |
| Energy | Electron <br> Volt | $1 \mathrm{eV}=1.602 \times 10^{-19} \mathrm{~J}$ |
| Force | Dyne | CGS unit $\left(\mathrm{g} \mathrm{cm} / \mathrm{s}^{2}=\mathrm{erg} / \mathrm{cm}\right)$ <br> 1 dyne $=10^{-5} \mathrm{~N}$ |
| Volume | Liter | $1 \mathrm{~L}=1 \mathrm{dm}^{3}$ |
| Pressure | Bar | $1 \mathrm{Bar}=10^{5} \mathrm{~Pa}$ |
| Length | Angstrom | $1 \AA=1 \times 10^{-10 \mathrm{~m}}$ |

## MATHEMATICAL CONCEPTS

A. Vector Algebra

1. Vector: Denotes directional character using $(x, y, z)$ components fig 1
a. Unit vectors: $\mathbf{i}$ along $x, \mathbf{j}$ along $y, \mathbf{k}$ along $z$
b. Vector $\boldsymbol{A}=A_{x} \mathbf{i}+A_{y} \mathbf{j}+A_{z} \mathbf{k}$
c. Length of $\boldsymbol{A}=|\boldsymbol{A}|=\sqrt{A_{x}^{2}+A_{y}^{2}+A_{z}^{2}}$
2. Addition of vectors $\boldsymbol{A} \boldsymbol{\&} \boldsymbol{B}$, add components:
$\boldsymbol{A}+\boldsymbol{B}=\left(A_{x}+B_{x}\right) \mathbf{i}+\left(A_{y}+B_{y}\right) \mathbf{j}+\left(A_{z}+B_{z}\right) \mathbf{k}$
Q Sample Addition and Length Calculations:

$$
A=3 \mathbf{i}+4 \mathbf{j}-3 \mathbf{k}
$$

$|A|=\sqrt{9+16+9}$
$B=-2 \mathbf{i}+6 \mathbf{j}+5 \mathbf{k}$

$$
=\sqrt{34}=5.83
$$

$|\boldsymbol{B}|=\sqrt{4+36+25}$

$$
=\sqrt{65}=8.06
$$

$\boldsymbol{A}+\boldsymbol{B}=\mathbf{i}+10 \mathbf{j}+2 \mathbf{k} \quad|\boldsymbol{A}+\boldsymbol{B}|=\sqrt{1+100+4}$
$=\sqrt{105}=10.25$
Note: $|\boldsymbol{A}|+|\boldsymbol{B}| \geq|\boldsymbol{A}+\boldsymbol{B}|$
3. Multiply $\boldsymbol{A} \& \boldsymbol{B}$ :
a. Dot or scalar product: $\boldsymbol{A} \cdot \boldsymbol{B}=|\boldsymbol{A}||\boldsymbol{B}| \cos \theta=$
$\left(A_{x} B_{x}\right)+\left(A_{y} B_{y}\right)+\left(A_{z} B_{z}\right)$
Note: $\theta$ is the angle between $\boldsymbol{A}$ and $\boldsymbol{B}$;
$\boldsymbol{A} \cdot \boldsymbol{B}=0$, if $\theta=\pi / 2$ fig 2
Sample: Scalar product:
$\boldsymbol{A}=5 \mathbf{i}+2 \mathbf{j} \quad \boldsymbol{B}=3 \mathbf{i}+5 \mathbf{j}$
$\boldsymbol{A} \cdot \boldsymbol{B}=3 \times 5+2 \times 5=15+10=25$
$|\boldsymbol{A}|=\sqrt{25+4}=\sqrt{29}=5.385$
$|\boldsymbol{B}|=\sqrt{9+25}=\sqrt{34}=5.831$
$\cos \theta=\frac{A \cdot B}{|A||B|}=\frac{25}{5.385 \times 5.83}=0.796$
$\theta=\cos ^{-1}(0.796)=37^{\circ}=0.2 \pi$ rad fig 3
b. Cross or Vector Product:
$\boldsymbol{C}=\boldsymbol{A} \times \boldsymbol{B}=|\boldsymbol{A}||\boldsymbol{B}| \sin \theta \mathbf{e}$
$\theta$ - Angle between $\boldsymbol{A}$ and $\boldsymbol{B}$, vector $\mathbf{e}$ is perpendicular to $\boldsymbol{A}$ and $\boldsymbol{B}$

$$
\boldsymbol{A} \times \boldsymbol{B}=\left|\begin{array}{ccc}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
A_{x} & A_{y} & A_{z} \\
B_{x} & B_{y} & B_{z}
\end{array}\right|
$$

© Sample: Vector Product:
$\boldsymbol{A}=2 \mathbf{i}+\mathbf{j} \quad \boldsymbol{B}=\mathbf{i}+3 \mathbf{j}$

$$
\left\lvert\, \begin{array}{lll}
\mathbf{i} & \mathbf{j} & \mathrm{k}
\end{array}\right.
$$

$\boldsymbol{A} \times \boldsymbol{B}=\begin{array}{lll}2 & 1 & 0\end{array}=(6-1) \mathbf{k}=5 \mathbf{k}$
$\left.1 \begin{array}{ll}1 & 3\end{array} \right\rvert\,$

- If $\boldsymbol{A}$ and $\boldsymbol{B}$ are in $x-y$ plane, $\boldsymbol{A} \times \boldsymbol{B}$ is along the $+z$ direction
$\cdot \theta$ is the angle formed by $\boldsymbol{A} \boldsymbol{B}: \sin \theta=\frac{|C|}{|A||B|}$ Given: $|\boldsymbol{A}|=\sqrt{5} \quad|\boldsymbol{B}|=\sqrt{10} \quad|\boldsymbol{C}|=5$ $\sin \theta=5 /(\sqrt{5} \times \sqrt{10})=5 / \sqrt{50}=1 / \sqrt{2}$

$$
<A B: \theta=45^{\circ}=\pi / 2 \text { radians }
$$

c. The Right-Hand Rule gives the orientation of vector e fig 4
B. Trigonometry

1. Basic relations for a triangle fig 5
$\sin \theta=\frac{y}{r} \quad$ Values Of $\sin , \cos$ and $\tan$
$\cos \theta=\frac{x}{r}$
$\tan \theta=\frac{y}{x}=\frac{\sin }{\cos }$
$\sin ^{2}+\cos ^{2}=1$
2. Sinand Cos waves
fig 6

| $\theta \mathrm{rad}\left[{ }^{\circ}\right]$ | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ |
| :---: | :---: | :---: | :---: |
| $0\left[0^{\circ}\right]$ | 0.00 | 1.00 | 0.00 |
| $\pi / 6\left[30^{\circ}\right]$ | 0.50 | 0.866 | 0.577 |
| $\pi / 4\left[45^{\circ}\right]$ | 0.707 | 0.707 | 1.00 |
| $\pi / 3\left[60^{\circ}\right]$ | 0.866 | 0.50 | 1.732 |
| $\pi / 2\left[90^{\circ}\right]$ | 1.00 | 0.00 | $\infty$ |
| $\pi\left[180^{\circ}\right]$ | 0.00 | -1.00 | 0.00 |


C. Geometry

Circle: Area $=\pi r^{2} ;$ Circumference $=2 \pi r$
Sphere: Volume $=4 / 3 \pi r^{3}$; Area $=4 \pi r^{2}$
Cylinder: Volume $=h \pi r^{2}$
Triangle: Sum of angles $=180^{\circ}$ fig 7

## D. Coordinate Systems

1. One dimension (1-D): position $=x$ fig 8

- The $x$ position is described relative to an origin

2. Two dimensions (2-D) fig 9
$x=r \cos \theta, y=r \sin \theta, r^{2}=x^{2}+y^{2}$
a. Calculate $(r, \theta)$ from $(x, y)$ :
$r=\sqrt{x^{2}+y^{2}} ; \quad \theta=\sin ^{-1}\left(\frac{y}{r}\right)$

b. Calculate $(x, y)$ from $(r, \theta)$, or $x$ and $y$ components of a vector " $r$ " with angle $\theta$; $x=r \cos \theta ; y=r \sin \theta$
Q Sample: Generate $x$ and $y$ vector
components, given: $r=5.0, \theta=\frac{\pi}{6}\left(30^{\circ}\right)$
$x=r \cos \left(\frac{\pi}{6}\right)=5.0 \times 0.866=4.33$
$y=r \sin \left(\frac{\pi}{6}\right)=5 \times 0.50=2.50$
Check your work: $x^{2}+y^{2}=r^{2}$
$2.5^{2}+4.33^{2}=6.25+18.75=25.00$
It checks, $r^{2}=25.00$
3. Three Dimensions (3-D)
a. Cartesian $(x, y, z)$ : The basic coordinate system
b. Cylindrical: $(r, \theta, z)$ fig 10

- Polar coordinates, with a $z$ axis
- Calculate $(r, \theta)$ from $(x, y)$; calculate $(x, y)$ from $(r, \theta)$
-Same process as for 2-d polar; $z$ : same as Cartesian
c. Spherical: $(r, \theta, \varphi)$
$x=r \sin \varphi \cos \theta, y=r \sin \varphi \sin \theta$,
$z=r \cos \varphi, r^{2}=x^{2}+y^{2}+z^{2}$ fig 11
- Calculate $(r, \theta, \varphi)$ from $(x, y, z)$
- Calculate $(x, y, z)$ from $(r, \theta, \varphi)$

Hint: Follow the strategy for 2-d polar coordinates
E. Use of Calculus in Physics

1. Methods from calculus are used in physics definitions, and the derivations of equations and laws
Physical meanings of calculus expressions:

a. Derivative - slope of the curve: $\frac{d F(x)}{d x} \quad x=r \sin \varphi \cos \theta$,
b. Integral - area under the curve: $\int F(x) \mathrm{d} x$

Q Samples:
$y=r \sin \varphi \sin \theta$,
$r^{2}=x^{2}+y^{2}+z^{2}$

- Position: $x$ or $F(x)$

Velocity: $v(x)=\frac{d F(x)}{d t}$
Acceleration: $a=\frac{d v(x)}{d t}$

- Power and work:
$P=\frac{d W}{d t}$
- Energy and force: $E=\int F \mathrm{~d} x$

2. Other useful expressions:
a. $\frac{d(F \cdot G)}{d x}=F \frac{d G}{d x}+G \frac{d F}{d x}$
b. $\frac{d(F \div G)}{d x}=\frac{1}{G} \frac{d F}{d x}-\frac{F}{G^{2}} \frac{d G}{d x}$
c. Partial derivative:
$\frac{\partial F(x, y, z)}{\partial x}=\frac{d F}{d x}$,
hold $y \& z$ constant
d. Gradient Operator $\boldsymbol{\nabla}$ (Del) $\partial / \partial x+\partial / \partial y+\partial / \partial z$
e. Integration by parts:
$\int u \mathrm{~d} v=u v-\int v \mathrm{~d} u$
f. Symbol for integration of closed surface or volume: $\oint$

Common derivatives and integrals

| $F(x)$ | $\frac{d F(x)}{d x}$ | $\int F(x) \mathrm{d} x$ |
| :---: | :---: | :---: |
| constant | 0 | constant $x$ |
| $x$ | 1 | $\frac{1}{2} x^{2}$ |
| $x^{2}$ | $2 x$ | $\frac{1}{3} x^{3}$ |
| $x^{n}$ | $n x^{n-1}$ | $\frac{1}{n+1} x^{n+1}$ |
| $\frac{1}{x}$ | $\frac{-1}{x^{2}}$ | $\ln x$ |
| $\ln x$ | $\frac{1}{x}$ | $x \ln x-x$ |
| $e^{x}$ | $e^{x}$ | $e^{x}$ |
| $\sin (x)$ | $\cos (x)$ | $-\cos (x)$ |
| $\cos (x)$ | $-\sin (x)$ | $\sin (x)$ |

PHYSICS \& MEASUREMANT
A. Understand Your Data

1. Vector vs. scalar
a. Vector: Has magnitude and direction
b. Scalar: Magnitude only, no direction
2. Number and unit
a. Physical data, constants and equations have numerical values and units
b. A correct answer must include the correct numerical value PLUS the correct unit
3. Significant figures (sigfig)
a. The \# of sigfigs reflects the accuracy of experimental data; calculations must accommodate this uncertainty
b. For multiplication: The \# of sigfigs in the final answer is limited by the entry with the fewest sigfigs
c. For addition: The \# of decimal places in the final answer is given by the entry with the fewest decimal places
d. Rules for "rounding sigfigs"

- If the last digit is $>5$, round up
- If the last digit is $<5$, round down
- If digit $=5$, round up if preceding digit is odd

Q Samples:
$1.245+0.4=1.6$ ( 1 decimal place )
$1.345 \times 2.4=3.2(2$ sigfigs $)$
Units of basic variables

| time: second s | position: meter m |
| :--- | :--- |
| mass: kilogram kg | volume: $\mathrm{m}^{3}$ |
| density: $\mathrm{kg} / \mathrm{m}^{3}$ | Temperature: K |
| velocity: $\mathrm{m} / \mathrm{s}$ | acceleration: $\mathrm{m} / \mathrm{s}^{2}$ |
| energy: Joule $\mathrm{J}=\mathrm{kg} \mathrm{m}^{2} / \mathrm{s}^{2}$ | force: Newton $\mathrm{N}=\mathrm{kg} \mathrm{m} / \mathrm{s}^{2}$ |

Pitfall: If the units are wrong, the answer is wrong!

## Hint: Before doing the calculation:

* Check all constants and variable units
* Take special care if you derive the equation

4. Dimensional analysis

Verify that constants and variables in an equation result in the correct overall unit
Q Samples: The energy unit is Joules for kinetic, gravitational and Coulombic energy

- Kinetic: $K=\frac{1}{2} m v^{2}$
$m$ in $\mathrm{kg}, v$ in $\mathrm{m} / \mathrm{s}$
Units of $K=\mathrm{kg}$
$-m \rightarrow v 12$ $\mathrm{m}^{2} / \mathrm{s}^{2}=J$ fig 12
-Gravitational potential energy:
$U_{\mathrm{g}}=m g h \quad$ Constant: $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$ $m$ in $\mathrm{kg}, h$ in $m$
Units of $U_{\mathrm{g}}=\mathrm{kg} \mathrm{m}^{2} / \mathrm{s}^{2}=J \quad$ fig 13



## MECHANIGS

A. Motion along a Straight Line

1. Goal: Determine position, velocity, acceleration
2. Key terms: Acceleration: $a=\mathrm{d} v / \mathrm{d} t$; velocity: $v=\mathrm{d} x / \mathrm{d} t$
3. Key Equations: $x=v_{i} t+\frac{1}{2} a t^{2}$
4. Key Equations: $x=v_{i} t+\frac{1}{2} a t^{2} \quad v_{f}=v_{i}+a t$
$\boldsymbol{x}(\boldsymbol{t}), \boldsymbol{v}(\boldsymbol{t})$ for variable $\boldsymbol{a}$ fig 15



$v_{i}$
$a=0$
~
$a>0$




- Electrostatic potential energy:

$$
U_{\mathrm{c}}=1 /\left(4 \pi \varepsilon_{0}\right) \frac{q_{1} q_{2}}{r} \quad \begin{gather*}
\boldsymbol{q}_{1} \ldots \tag{14}
\end{gather*} \ldots \boldsymbol{q}_{2}
$$

constant: $1 / 4 \pi \varepsilon_{0}$; units are $\mathrm{J} \mathrm{m} / \mathrm{C}^{2}$
$r$ in $\mathrm{m}, q_{1}$ and $q_{2}$ in Coulomb
Units of $U_{\mathrm{c}}=\left(\mathrm{J} \mathrm{m} / \mathrm{C}^{2}\right) \mathrm{C}^{2} / \mathrm{m}=\mathrm{J}$ fig 14

## 5. Using Conversion Factors

a. Purpose: Modify experimental data to match the units of constants and equations
b. SI units: MKS (m-kg-s) and CGS (cm-g-s)
c. Common English units: Foot, pounds,

BTU, calories
d. Conversion factors are obtained from an equality of two units
จample: $100 \mathrm{~cm}=1 \mathrm{~m}$

- This equality gives two conversion factors: $\frac{1 \mathrm{~m}}{100 \mathrm{~cm}} \& \frac{100 \mathrm{~cm}}{1 \mathrm{~m}}$
- Use the $1^{\text {st }}$ factor to convert "cm" to " $m$ "

Q Sample: $54 \mathrm{~cm} \times \frac{1 \mathrm{~m}}{100 \mathrm{~cm}}=0.54 \mathrm{~m}$

- Use the $2^{\text {nd }}$ to convert " $m$ " to " cm "

2. Sample: $2.3 \mathrm{~m} \times \frac{100 \mathrm{~cm}}{1 \mathrm{~m}}=230 \mathrm{~cm}$
B. Solve the Problem Strategically
a. Two key issues:
3. Understand the physics principles
4. Have a correct mathematical strategy
b. Useful steps in problem solving:
5. Prepare a rough sketch of the problem
6. Identify relevant physical variables, physical concepts and constants
7. Pitfall - do not simply search for the "right" equation in your notes or text
8. Describe the physics using a mathematical diagram, with appropriate symbols and a coordinate system
9. Obtain the relevant physical constants Do you have all the essential data?
Hint: You may have extra information
10. The hard part: Derive or obtain a mathematical expression for the problem; use dimensional analysis to check the equation, constants and data
11. The easy part: Plug numbers into the equation and use the calculator to obtain the numerical answer
12. Check the final answer, using the original statement of the problem, your sketch and common sense; are the units \& sign correct?
B. Motion in Two and Three Dimensions
13. Goal: Similar to " $\boldsymbol{A}$," with 2 or 3 dimensions
14. Key concept: Select Cartesian, polar or spherical coordinates, depending on the type of motion
Sample: A projectile is launched at angle $\theta$ with $v_{r i}$, how do we set up the problem?
Step 1. Define $x$ as horizontal and $y$ vertical
Step 2. Determine initial $v_{x i}$ and $v_{y i}$ fig 16
$v_{x i}=v_{r i} \cos \theta$
$v_{y i}=v_{r i} \sin \theta$


Step 3. Identify $a_{x}$ - Gravitational force $\Rightarrow a_{y}=-g$

Step 4. Identify $a_{y}$ - No horizontal force $\Rightarrow a_{x}=0$
Step 5. Develop $x$ - and $y$-equations of motion

$$
\begin{aligned}
& x=v_{i x} t+\frac{1}{2} a_{x} t^{2}=v_{i} t \\
& y=v_{i y} t+\frac{1}{2} a_{y} t^{2}=v_{i y} t-\frac{1}{2} g t^{2}
\end{aligned}
$$

C. Newton's Laws of Motion

1. Goal: Examine force and acceleration
2. Key concepts: Newton's Laws:

Law \#1. A body remains at rest or in motion unless influenced by a force
Law \#2. Forces acting on a body equal the mass multiplied by the acceleration; force and acceleration determine motion
Law \#3. Every action is countered by an opposing action
3. Key equations:
a. Law \#2: $\boldsymbol{F}=m \boldsymbol{a}$ or $\Sigma \boldsymbol{F}=m \boldsymbol{a}$

Hint: Forces are vectors!
b. Types of forces: Body - gravity: $F_{g}=m \boldsymbol{g}$

- Surface - friction: $=F_{f}=\mu F_{n}$

Q Sample: $F_{t}$ exerted on object on a horizontal plane $F_{f}=\mu F_{n}=\mu F_{g}=\mu m g$ Net force $=F_{t}-F_{f}$ fig 17
Qample: Object on plane inclined at angle $\theta$;
examine $F_{g} \& F_{f}$
$F_{n}=F_{g} \cos \theta=m g \cos \theta$
$F_{f}=\mu F_{n}=\mu m g \cos \theta$
$F_{t}=m g \sin \theta$ fig 18
e. Law \#3:
$F_{12}=-F_{21}$ or $m_{1} a_{1}=-m_{2} a_{2}$
Q Sample: Examine recoil of bullet fired from a rifle


Rifle recoil $=a$ (bullet) $\times m$ (bullet)
D. Circular Motion fig 19

1. Goal: Examine body moving in a circular path; use 2-d polar coordinates: $(r, \theta)$

## Key variables:

| $r$ | m | distance from <br> rotation center |
| :---: | :---: | :--- |
| $\theta$ | rad | angle with <br> reference $(x)$ axis |
| $\omega$ | $\mathrm{rad} / \mathrm{s}$ | angular velocity |
| $\alpha$ | $\mathrm{rad} / \mathrm{s}^{2}$ | angular <br> acceleration |
| $s$ | m | motion arc; <br> $s=r \theta(\theta$ in rad $)$ |



Hint: For a full rotation, $s=2 \pi r=$ circumference of a circle of radius $r$
2. Tangential acceleration \& velocity: $v_{t}=r \omega ; a_{t}=r \alpha$; along path of motion arc
3. Centripetal acceleration: $a_{c}=\frac{v^{2}}{r}$; directed towards the center fig 19
Q Sample: Determine $v_{t}$ at the Earth's equator
Equation: $v_{t}=r \omega \quad$ Data: $r=6.378 \times 10^{6} \mathrm{~m}$
$\omega=2 \pi \mathrm{rad} /$ day; $\quad 1$ day $=24 \times 60 \times 60 \mathrm{sec}=86,400 \mathrm{~s}$
Convert $\omega$ to SI: $\omega=2 \pi \mathrm{rad} /$ day $\times 1$ day $/ 86,400 \mathrm{~s}=$ $7.3 \times 10^{-5} \mathrm{rad} / \mathrm{s}$
Calculate $\boldsymbol{v}_{\boldsymbol{t}}$ :
$v_{t}=r \omega=6.378 \times 10^{6} \mathrm{~m} \times 7.3 \times 10^{-5} \mathrm{rad} / \mathrm{s} v_{t}=465 \mathrm{~m} / \mathrm{s}$
E. Energy and Work

1. Goal: Examine the energy and work associated with forces acting on an object
2. Key equations:
a. Kinetic energy: $\frac{1}{2} m v^{2}$; energy of motion
b. Work: Force acting over a distance

- For $F(x)$ : Work $=\int F(x) d x$
- For a constant force: $W=F d \cos \theta=\boldsymbol{F} \times \boldsymbol{r}$ - $\theta$ is the angle between the $F$ and $r$
- $W$ maximum for $\theta=0($ note: $\sin (\theta=0)=1)$
c. Power $=$ Work/time: $W=$ Power $\Delta t$ or $\int P(t) d t$
d. $\boldsymbol{W}_{\text {net }}=\boldsymbol{K}_{\text {final }}-\boldsymbol{K}_{\text {initial }} ; K$ is converted to work
* Sample: Determine the work expended in lifting
a 50 kg box 10 m ; given: $a=g=9.8 \mathrm{~m} / \mathrm{s}^{2}$
Equations: $F=m g \Rightarrow \mathrm{~W}=m g d$
Calculation: $W=50 \mathrm{~kg} \times 9.8 \mathrm{~m} / \mathrm{s}^{2} \times 10 \mathrm{~m}=4,900 \mathrm{~J}$
F. Potential Energy \& Energy Conservation

1. Goal: Use energy conservation to study the interplay of potential and kinetic energy
2. Key Equations
a. Potential energy: Energy of position: $U(r)$;
gravitation $(U=m g h)$,
electrostatic ( $U \alpha q q / r$ )
b. $\boldsymbol{E}=\boldsymbol{K}+\boldsymbol{U}$ Conservative system: No external force

Sample: Examine $K \& U$ for a launched rocket
Initial: $h=0$, therefore, $U=m g h=0$ (1)
$E=K_{i}=\frac{1}{2} m v_{i}^{2}$
Next, resolve into $x$ and $y$ components: $K_{x i} \& K_{y i}$
Note: $K_{x}$ is constant during
the flight
At max height: $K_{y}=0 ; U=$
$m g h=K_{x i}{ }^{(2)}$
Final state: Rocket hits the ground: $U=0, K=K_{i}(3$
 fig 20
G. Collisions and Linear Momentum fig 21

1. Goal: Examine momentum of colliding bodies

Hint: For 2-D or 3-D, use
Cartesian components
2. Key Variables and Equations
a. Types of collisions:

- Elastic: Conserve energy
- Inelastic: Energy lost as heat or deformation
b. Relative motion and frames of reference: A body moves with velocity $v$ in frame $S$; in frame $S^{\prime}$, the velocity is $v^{\prime}$; if $V_{s^{\prime}}$ is the velocity of frame $S^{\prime}$ relative to $S$, then $\boldsymbol{v}=\boldsymbol{V}_{\boldsymbol{s}},+\boldsymbol{v}$,
c. Linear Momentum: $\boldsymbol{p}=\boldsymbol{m} \boldsymbol{v}$
d. Conserve $K \& \boldsymbol{p}$ for conservative system (no external forces):

$$
\Sigma \frac{1}{2} m v_{i}^{2}=\Sigma \frac{1}{2} m v_{f}^{2} \quad \Sigma m v_{i}=\Sigma m v_{f}
$$

Q Sample 1-d problem: Two bodies collide, stick together and move away from the collision site fig 22 Conservation of momentum: $m_{1} v_{1 i}+m_{2} v_{2 i}=\left(m_{1}+m_{2}\right) v_{f}$

f. Impulse: $I=F \Delta t$ or $\int F(t) d t$
g. Momentum change: $p_{\text {fin }}=p_{\text {init }}+I$
H. Rotation of a Rigid Object

1. Goal: Examine the rotation of a rigid body of a defined shape and mass
2. Key variables and equations:
a. Center of mass: $x_{c m}, y_{c m}, z_{c m}$
$x_{c m}=\frac{\sum m_{i} x_{i}}{\sum m_{i}} \quad y_{c m}=\frac{\sum m_{i} y_{i}}{\sum m_{i}} \quad z_{c m}=\frac{\sum m_{i} z_{i}}{\sum m_{i}}$
Q Sample: Calculate the center of mass for a 1 kg \& a 2 kg ball connected by a 1.00 m bar ball 1: $x_{1}=0.00, m_{1}=1 \mathrm{~kg} ; m_{1} x_{1}=0.00 \mathrm{~kg} m$ ball 2: $x_{2}=1.00 m_{2}=2 \mathrm{~kg} ; m_{2} x_{2}=2.00 \mathrm{~kg} m$ $\Sigma m_{i}=1 \mathrm{~kg}+2 \mathrm{~kg}=3 \mathrm{~kg}$ $\Sigma m_{i} x_{i}=m_{1} x_{1}+m_{2} x_{2}=0.00+2.00=2.00 \mathrm{~kg} \mathrm{~m}$ $\boldsymbol{x}_{\boldsymbol{c m}}=\frac{\sum m_{i} x_{i}}{\sum m_{i}}=\frac{2.00 \mathrm{~kg} \mathrm{~m}}{3.00 \mathrm{~kg}}=0.66 \mathrm{~m}$
Hint: The center of mass is nearer the heavier ball fig 23

b. Moment of inertia:
$I=\Sigma m_{i} r_{i}{ }^{2}$, with $r_{i}$ about the center of mass along a specific axis

Hint: $I$ functions as the effective mass for rotational energy and momentum
Qample: I for bodies of mass $m$ : fig 24
Twirling thin rod of length, $L$ $\mathrm{I}=\frac{1}{12} m L^{2}$
Rotating cylinder of radius, $R$

$$
I=\frac{1}{2} m R^{2}
$$

Rotating sphere of radius, $R$

$$
I=\frac{2}{5} m R^{2}
$$



Q Sample: Determine the $I$ for a spherical Earth, assume uniform $M$;
Data: $M=6 \times 10^{24} \mathrm{~kg}, r=6.4 \times 10^{6} \mathrm{~m}$
$I=\frac{2}{5} M r^{2}=\frac{2}{5} \times 6 \times 10^{24} \mathrm{~kg} \times\left(6.4 \times 10^{6} \mathrm{~m}\right)^{2}$
$=9.8 \times 10^{37} \mathrm{~kg} \mathrm{~m}^{2}$
e. Rotational Energy $=\frac{1}{2} I \omega^{2}$
f. Torque: $\boldsymbol{\tau}=I \omega=\boldsymbol{r} \times \boldsymbol{F}$ (ang. acceleration force)

## I. Angular Momentum

1. Goal: Quantify the force, energy
and momentum of rotating objects
2. Key variables and equations
a. Angular momentum:
$\boldsymbol{L}=I \omega=\boldsymbol{r} \times \boldsymbol{p}=\int \boldsymbol{r} \times \boldsymbol{v} d m$
b. Torque: $\boldsymbol{\tau}=\boldsymbol{r} \times \boldsymbol{F}=d \boldsymbol{L} / d t$; note: vector cross product fig 25
J. Static Equilibrium and Elasticity
3. Case 1: Examine several forces acting on a body

- Guiding principles: Equilibrium is defined as: $\Sigma$ force $=0 \& \Sigma$ torque $=0$
The point of balance is the center of mass
O Hint: Evaluate each component; any net force moves the object, any net torque rotates the object
Q Sample: Beam balance fig 26


Note: Force $\boldsymbol{F}_{1}$ is longitudinal

- Shape Stress: Shear Modulus $S$
$\boldsymbol{S}=\frac{F_{t} / A}{\Delta x / h}$
fig $27 b$


Note: Force $F_{t}$ is tangential to face $A$

- Volume Stress: Bulk Modulus $B$

$$
B=\frac{F_{n} / A}{\Delta V / V}
$$

fig 27c


Note: Force $F_{n}$ is normal to face $A$

## MECHANICS (continued)

## K. Universal Gravitation

1. Goal: Examine gravitational energy and force fig 28
2. Case 1: Bodies of mass $M_{1} \& M_{2}$ separated by $r$
3. Key equations:
a. Gravitational Energy: $U_{g}=\frac{G M_{1} M_{2}}{r}$
b. Gravitational force: $F_{g}=\frac{G M_{1} M_{2}}{r^{2}}$
c. Acceleration due to gravity: $\boldsymbol{g}=\boldsymbol{G} \boldsymbol{M}($ earth $) / \boldsymbol{r}^{2}$


For objects on the Earth's surface, $g=9.8 \mathbf{~ m} / \mathbf{s}^{2}$
Q Sample: Verify " $g$ " at the Earth's surface
Equation: $g=G M($ earth $) / r^{2}$
Given: $\mathrm{M}=6 \times 10^{24} \mathrm{k}, r=6.4 \times 10^{6} \mathrm{~m}$
Calculation: $=\frac{6.67 \times 10^{-11} \mathrm{~m}^{3} \mathrm{~kg}^{-1} \times 6 \times 10^{24} \mathrm{~kg}}{\left(6.4 \times 10^{6} \mathrm{~m}\right)^{2}}=\mathbf{9 . 8} \mathbf{m s}^{2}$
4. Case 2: A body interacts with the Earth fig 29

## 5. Key Equation:

a. Gravitational potential energy: $U_{g}=m g h$; object on the Earth's surface, $h=0 ; U_{g}=0$
b. Weight = gravitational force; $F_{g}=m g$

Q Sample: Calculate escape velocity, $v$ esc , for an orbiting rocket of mass $m$ at altitude $h$

Hint: $K=U_{g}$ at point of escape; $r=h+r$ (earth) $\frac{1}{2} m v_{\mathrm{esc}}{ }^{2}=\frac{G m M}{r}$; therefore, $v_{\mathrm{esc}}=\sqrt{\frac{2 G M}{r}}$
Note: $v_{\text {esc }}$ varies with altitude, but not rocket mass

## L. Oscillatory Motion

1. Goal: Study motion \& energy of oscillating body
2. Simple harmonic motion (1-d)
a. Force: $F=-k \Delta x$ (Hooke's Law)
b. Potential Energy: $U_{k}=\frac{1}{2} k \Delta x^{2}$
c. Frequency $=\frac{1}{2 \pi} \sqrt{\frac{k}{m}} \quad$ fig 30

3. Simple Pendulum
a. Period: $T=2 \pi \sqrt{\frac{l}{g}}$

4. For both cases:
a. Kinetic energy: $K=\frac{1}{2} m v^{2}$
b. Conservation of Energy: $E=U+K$

## M. Forces in Solids and Liquids

1. Goal 1: Examine properties of solids \& liquids
a. Density of a solid or liquid: $\boldsymbol{\rho}=\frac{\text { mass }}{\text { volume }}$
-Common unit: $\mathrm{g} / \mathrm{cm}^{3} ; \mathrm{g} / \mathrm{L} ; \mathrm{kg} / \mathrm{m}^{3}$

-     - Sample: A piece of metal, $1.5 \mathrm{~cm} \times 2.5 \mathrm{~cm} \times 4.0 \mathrm{~cm}$, has a mass of 105.0 g ; determine $\rho$
Equation: $\rho=\frac{m}{V}$
Data: $m=105.0 \mathrm{~g}, V=1.5 \times 2.5 \times 4.0 \mathrm{~cm}^{3}=15 \mathrm{~cm}^{3}$
Calculate: $\rho=105.0 / 15.0 \mathrm{~g} / \mathrm{cm}^{3}=7.0 \mathrm{~g} / \mathrm{cm}^{3}$
b. Pressure exerted by a fluid: $P=\frac{\text { force }}{\text { area }}$
c. Pascals's Law: For an enclosed fluid, pressure is equal at all points in the vessel . Sample: Hydraulic press: $F=P / A$ for enclosed liquid; $A$ is the surface area of the piston inserted into the fluid
Equation: $A_{1} F_{1}=A_{2} F_{2}$; cylinder area determines force fig 32
d. A column of water generates pressure, $P$ increases with depth;

Equation: $P_{2}=P_{1}+\rho g h$ fig 33
e. Archimedes' Principle: Buoyant force, $F_{b}$, on a object of volume $V$ submerged in liquid of density $\rho: F_{b}=\rho V g$ fig 34
2. Goal 2: Examine fluid motion \& fluid dynamics
a. Properties of an Ideal fluid: Non-viscous, incompressible, steady flow, no turbulence At any point in the flow, the product of area and velocity is constant: $\boldsymbol{A}_{1} \boldsymbol{v}_{1}=\boldsymbol{A}_{2} \boldsymbol{v}_{2}$
b. Variable density: $\rho_{1} \boldsymbol{A}_{1} \boldsymbol{v}_{1}=\rho_{2} A_{2} v_{2}$; illustrations: gas flow through a smokestack, water flow through a hose fig 35
c. Bernoulli's Equation: For any point $y$ in the fluid flow, $P+\frac{1}{2} \rho v^{2}+\rho g y=$ constant - Special case: Fluid at rest $P_{1}-P_{2}=\rho g h$

## WAVE MOTION

## A. Descriptive Variables

1. Types: Transverse, longitudinal, traveling, standing, harmonic
a. General form for transverse traveling wave: $y=f(x-v t)$ (to the right) or $y=f(x+v t)$ (to the left)
b. General form of harmonic wave: $y=A \sin (k x-\omega t)$ or $y=A \cos (k x-\omega t)$
c. Standing wave: Integral multiples of $\frac{\lambda}{2}$ fit the length of the oscillating material
d. General wave equation: $\frac{d^{2} y}{d x^{2}}=\frac{1}{v^{2}} \frac{d^{2} y}{d t^{2}}$
e. Superposition Principle: Overlapping waves interact $=>$ constructive and destructive interference
Harmonic Wave Properties

| Wavelength | $\lambda(\mathrm{m})$ | Distance between peaks |
| :--- | :---: | :--- |
| Period | $T(\mathrm{sec})$ | Time to travel one $\lambda$ |
| Frequency | $f(\mathrm{~Hz})$ | $f=\frac{1}{T}$ |
| Angular Frequency | $\omega(\mathrm{rad} / \mathrm{s})$ | $\omega=\frac{2 \pi}{T}=2 \pi f$ |
| Wave Amplitude | $A$ | Height of wave |
| Speed | $v(\mathrm{~m} / \mathrm{s})$ | $v=\lambda f$ |
| Wave number | $k\left(\mathrm{~m}^{-1}\right)$ | $k=\frac{2 \pi}{\lambda}$ |

2. Sample: Determine the velocity and period of a wave with
$\lambda=5.2 \mathrm{~m}$ and $f=50.0 \mathrm{~Hz}$
Equations: $v=\lambda f \quad T=\frac{1}{f}$
Data: $\lambda=5.20 \mathrm{~m} ; f=50.0 \mathrm{~Hz}$
Calculations: $v=\lambda f=5.20 \mathrm{~m} \times 50.0=260 \mathrm{~m} / \mathrm{s}$
$T=\frac{\mathbf{1}}{\boldsymbol{f}}=\frac{1}{50} \mathrm{~Hz}=0.02 \mathrm{~s}$

## B. Sound Waves

1. Wave nature of sound: Compression wave displaces the medium carrying the wave
2. General speed of sound: $v=\sqrt{\frac{B}{\rho}}$;
note: $B=$ Bulk Modulus (measure of volume compressibility) For a gas: $v=\sqrt{\frac{\gamma R T}{M}}$; note: $\gamma=\frac{C_{p}}{C_{v}}$ (ratio of gas heat capacities)
Q Sample: Calculate speed of sound in Helium at 273 K
Helium: Ideal gas, $\gamma=1.66 ; M=0.004 \mathrm{~kg} / \mathrm{mole}$
$v=\sqrt{\frac{\gamma R T}{M}}$
$=\sqrt{\frac{1.66 \times 8.314 \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}^{2} \times 273 \mathrm{~K}}{0.004 \mathrm{~kg}}}$
$=\sqrt{941,900 \mathrm{~m}^{2} / \mathrm{s}^{2}}=971 \mathrm{~m} / \mathrm{s}$ note: $\sqrt{ }$ applies to the units
3. Loudness as intensity and relative intensity
a. Absolute Intensity ( $I=$ Power/Area) is an inconvenient measure of loudness
b. Relative loudness: Decibel scale (dB): $\beta=10 \log \frac{I}{I_{0}} ; I_{0}$ is the threshold of hearing; $\beta\left(I_{0}\right)=0$
c. Samples: Jet plane: 150 dB ; Conversation: 50 dB ; a change in 10 dB represents a 10-fold increase in $I$
4. Doppler effect: The sound frequency shifts $\frac{f^{\prime}}{f}$ due to relative motion of source and listener;
$v_{0}$ - listener speed; $v_{\mathrm{S}}$ - source speed; $v$ - speed of sound

$\frac{f^{\prime}}{f}=\frac{v-v_{0}}{v+v_{s}}$

Key: Identify relative speed of source and listener

## THERMODYNAMICS

A. Goal: Study of work, heat and energy of a system fig 36 Key Variables

| Heat: $Q$ | $+Q$ added to the system |
| :--- | :--- |
| Work: $W$ | $+W$ done by the system |
| Energy: $E$ | System internal $E$ |
| Enthalpy: $H$ | $H=E+P V$ |
| Entropy: $S$ | Thermal disorder |
| Temperature: $T$ | Measure of thermal $E$ |
| Pressure: $P$ | Force exerted by a gas |
| Volume: $V$ | Space occupied |



## THERMODYNAMICS (continued)

Types of Processes

| Isothermal | $\Delta T=0$ | $\Delta E=0, Q=W$ <br> $P V=$ constant |
| :--- | :--- | :--- |
| Adiabatic | $Q=0$ | $\Delta E=-W$ <br> $P V \gamma=$ constant |
| Isobaric: | $\Delta P=0$ | $\mathrm{W}=P \Delta V$, <br> $\Delta H=Q$ |
| Isochoric | $\Delta V=0$ | $\Delta E=Q ;$ <br> $W=0$ |

B. Temperature \& Thermal Energy

1. Goal: Temperature is in Kelvin, absolute temperature: $\boldsymbol{T}(\mathrm{K})=\boldsymbol{T}\left({ }^{\circ} \mathrm{C}\right)+273.15$
Note: $T(\mathrm{~K})$ is always positive; lab temperature must be converted from ${ }^{\circ} \mathrm{C}$ to Kelvin (K)
Q Sample: Convert $35^{\circ} \mathrm{C}$ to Kelvin:
$T(\mathrm{~K})=T\left({ }^{\circ} \mathrm{C}\right)+273.15=35+273.15=$ 308.15 K
2. Thermal Expansion of Solid, Liquid or Gas
a. Goal: Determine the change in the length $(L)$ or volume $(V)$ as a function of temperature
b. Solid: $\frac{\Delta L}{L}=\alpha \Delta T$
c. Liquid: $\frac{\Delta V}{V}=\beta \Delta T$
d. Gas: $\Delta V=\frac{\left(T_{2}-T_{1}\right) n R}{P}$
3. Heat capacity: $\boldsymbol{C}=\frac{Q}{\Delta T}$ or $\boldsymbol{Q}=\boldsymbol{C} \Delta \boldsymbol{T}$
a. Special cases: $\boldsymbol{C}_{\boldsymbol{p}}$-constant $P ; \boldsymbol{C}_{\boldsymbol{v}}$-constant $V$ -Ideal Gas:
$C_{\boldsymbol{p}}=\frac{5}{2} R ; C_{v}=\frac{3}{2} R ; \gamma=\frac{C_{p}}{C_{v}}=\frac{5}{3}=1.667$
b. Carnot's Law: For ideal gas: $C_{\boldsymbol{p}}-C_{v}=R$

- $\Delta E=C_{v} \Delta T ; \Delta H=C_{p} \Delta T$
- Exact for monatomic gas, modify for molecular gases

37

Boyle's Law


Pressure (Pa)

Charles' Law


Temperature (K) fig 37

1. Goal: Simple equation of state for a gas
2. Key Variables: $P(\mathrm{~Pa}), V\left(\mathrm{~m}^{3}\right), T(\mathrm{~K}), n$ moles of gas (mol); gas constant $R=8.314 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$ $\$$ Pitfall: Common errors in $T, P$ or $V$ units
3. Key Applications:
a. $P \propto \frac{1}{V}, T$ fixed: Boyle's Law
b. $P \propto T, V$ fixed
c. $V \propto T, P$ fixed: Charles' Law
d. Derive thermodynamic relationships
D. Enthalpy and $1^{\text {st }}$ Law of Thermodynamics
4. Goal: Determine $Q, \Delta E$ and $W ; W$ and $Q$ depend on path; $\Delta E$ is a state variable, independent of path

## 2. Guiding Principle:

a. $1^{\text {st }}$ Law of Thermodynamics: $\Delta E=\boldsymbol{Q}-\boldsymbol{W}$

- Key idea: Conservation of Energy
b. Examine the $T, P, W \& Q$ for the problem

3. Enthalpy: $\boldsymbol{H}=\boldsymbol{E}+\boldsymbol{P V} ; \Delta \boldsymbol{H}=\Delta \boldsymbol{E}+\boldsymbol{P} \Delta \boldsymbol{V}$
a. $\Delta H=Q$ for $\Delta P=0$ (constant pressure)
b. Variable temperature: $\Delta H=\int C_{p} d T$
c. For constant $C_{p}: \Delta H=C_{p} \Delta T$
4. Work: $W=\int P d V$
a. $W$ depends on the path or process
b. Ideal Gas, Reversible, Isothermal: $W=n R T \ln \frac{V_{2}}{V_{1}}$
c. Ideal Gas, Isobaric: $W=P \Delta V$
E. The Kinetic Theory of Gases
5. Goal: Examine kinetic energy of gas molecules
6. Key Equations: $\mathrm{E}=\frac{1}{2} M v^{2}$ and $E=\frac{3}{2} R T$
a. Speed, $v_{\mathrm{rms}}=\sqrt{\frac{3 R T}{M}}$

Q Sample: Calculate the speed of Helium at 273 K
Helium: $M=0.004 \mathrm{~kg} /$ mole
$v_{\mathrm{rms}}=\sqrt{\frac{3 R T}{M}}=$
$\sqrt{\frac{3 \times 8.314 \mathrm{~kg} \mathrm{~m}^{2} / \mathrm{s}^{2} \times 273 \mathrm{~K}}{0.004 \mathrm{~kg}}}$
$\boldsymbol{v}_{\mathrm{rms}}=\sqrt{1,702,292} \mathbf{m} / \mathbf{s}=\mathbf{1 3 0 5} \mathbf{~ m} / \mathbf{s}$
b. Kinetic energy for Ideal Gas: $K=\frac{3}{2} R T$
c. For real gas: Add terms for vibrations and rotations
F. Entropy \& $2^{\text {nd }}$ Law of Thermodynamics
. Goal: Examine the driving force for a process
2. Key Variables:
a. Entropy: $S$, thermal disorder; $d S=\frac{d Q}{T}$
b. $S($ univ $)=S($ system $)+S($ thermal bath $)$
3. Guiding Principle: $2^{\text {nd }}$ Law of

Thermodynamics:
For any process, $\Delta S_{\text {univ }}>0$; one exception: $\Delta S_{\text {univ }}=0$ for a reversible process

## 4. Examples:

a. Natural heat flow: $Q$ flows from $T_{\text {hot }}$ to $T_{\text {cold }}$ fig 38
$\Delta S_{\text {univ }}=\Delta S_{\text {hot }}+\Delta S_{\text {cold }}=$
$-\frac{Q}{T_{\text {hot }}}+\frac{Q}{T_{\text {cold }}}=Q \frac{T_{\text {hot }}-T_{\text {cold }}}{T_{\text {hot }} T_{\text {cold }}}$
hint: $\Delta S_{\text {univ }}>0$ for a
 natural process

## b. Phase change: $\Delta S=$ <br> Q(phase change) <br> $T$ (phase change)

c. Ideal Gas $S(T): \Delta S=n C_{p} \ln \frac{T_{2}}{T_{1}}$
d. Ideal Gas: $S(V): \Delta S=n R \ln \frac{V_{2}}{V_{1}}$
G. Heat Engines

1. Goal: Examine $Q$ and $W$ of an engine
2. Thermal Engine: The engine transfers $Q$ from a hot to a cold reservoir, and
 produces $W$ fig 39
3. Efficiency of
engine: $\xi=\frac{W}{Q_{\text {hot }}}=1-\frac{Q_{\text {cold }}}{Q_{\text {hot }}}$

4. Idealized heat engine: Carnot Cycle fig 40
a. Four steps in the cycle: two isothermal, two adiabatic; for overall cycle: $\Delta E=0$ and $\Delta S=0$
b. Efficiency $=1-\frac{T_{\text {cold }}}{T_{\text {hot }}}$
A. Electric Fields and Electric Charge
5. Goal: Examine the nature of the field generated by an electric charge, and forces between charges
6. Key Variables and Equations
a. Coulomb C: "ampere sec" of charge
b. $e$ - charge on an electron; $1.6022 \times 10^{-19} \mathrm{C}$
c. Coulomb' Law - electrostatic force: $F=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1} q_{2}}{r^{2}} \boldsymbol{e}$

- Vector direction defined by $\boldsymbol{e}$
d. Electric Field: $E=\frac{F}{q}$

Hint: Calculation shortcut:

$$
F=9 \times 10^{9} N \frac{q_{1}(C) q_{2}(C)}{r(m)^{2}}
$$

Note: $q$ in Coulombs and $r$ in meters
3. Superposition Principle: Forces and fields are composites of contributions from each charge
$F=\Sigma F_{i}, E=\Sigma E_{i} ; \bigcirc$ Hint: Forces and electric fields are vectors
B. Gauss's Law

1. Goal: Define electric flux, $\Phi_{e}$
2. Key Variables and Equations:
a. Gauss's Law: $\Phi_{e}=\oint E \times d A=\frac{Q}{\varepsilon_{0}}$
b. The electric flux, $\Phi_{e}$, depends on the total charge in the closed region of interest
C. Electric Potential \& Coulombic Energy
3. Goal: Determine Coulombic potential energy
4. Key Variables and Equations:
a. Potential energy: $U=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1} q_{2}}{r}$
b. Potential: $V\left(q_{1}\right)=\frac{U}{q_{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1}}{r}$

Note: The potential is scalar, depending on $|r|$
c. For an array of charges, $q_{i}, V_{\text {total }}=\Sigma V_{i}$
d. Shortcut to $\boldsymbol{U}(\boldsymbol{r}): U=9 \times 10^{9} \mathrm{~J} \frac{q_{1}(C) q_{2}(C)}{r(m)}$
3. Continuous charge distributions: $V=\frac{1}{4 \pi \varepsilon_{0}} \int \frac{d q}{r}$ fig 41

Q Sample: Conducting sphere, Radius $R$, Charge $Q$
$V=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{R}$ for $r \leq R$
$V=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r}$ for $r>R$
4. Dielectric effect: $V \& F$ depend on the dielectric constant, $\kappa$; replace $\varepsilon_{0}$ with $\kappa \varepsilon_{0}$ for the material;

$V(\kappa)=\frac{1}{\kappa} V($ vacuum $)$
Conducting Sphere
$\mathrm{F}(\kappa)=\frac{1}{\kappa} F($ vacuum $)$
D. Capacitance and Dielectrics

1. Goal: Study capacitors, plates with charge $Q$ separated by a vacuum or dielectric material fig 42

2. Key Equations:
a. Capacitance, $C=\frac{Q}{V}$, $V$ is the
measured voltage
b. Parallel plate capacitor, vacuum, with area $A$,
spacing $d$ : $C=\varepsilon_{0} \frac{A}{d} ; \boldsymbol{E}=\frac{Q}{\varepsilon_{0}} A$
c. Parallel plate capacitor, dielectric $\kappa$, with area $A$,
spacing d: $C=\kappa \varepsilon_{0} \frac{A}{d}$
3. Capacitors in series: $\frac{1}{C_{\text {tot }}}=\sum \frac{1}{C_{i}}$
4. Capacitors in parallel: $C_{\text {tot }}=\Sigma C_{i}$ fig 43

Two Capacitors in Series

Two Capacitors in Parallel
$C_{2} C_{1} C_{2} C_{\text {tot }}=C_{1}+C_{2}$

ELECTRICTY \& MACNETISM (continued)
E. Current and Resistance

1. Goal: Examine the current, $I$, quantity of charge, $Q$, resistance, $R$; determine the voltage and power dissipated 2. Key Equations:
a. Total charge, $Q=I t$
b. $V=I R$, or $R=\frac{V}{I}$
c. Resistors in Series: $R_{\text {tot }}=\Sigma R_{i} \quad$ fig 44 d. Resistors in Parallel:

Two Resistors in Series
 $\frac{1}{R_{\text {tot }}}=\Sigma \frac{1}{R_{i}} \quad$ fig 44 e. Power $=I V=I^{2} R$
F. Direct Current Circuit

Two Resistors in Parallel

$$
\begin{aligned}
& \text { Two Resistors in Parallel } \\
& \underbrace{R_{1}}\left\{^ { R _ { 2 } } \left\{\frac{1}{R_{\mathrm{tot}}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}\right.\right.
\end{aligned}
$$

1. Goal: Examine a circuit containing battery,
or $\quad R_{\text {tot }}=\frac{R_{1} R_{2}}{R_{1}+R_{2}}$ resistors and capacitors; determine voltage and current properties
2. Key Equations and Concepts:
a. EMF: Circuit voltage; $\mathrm{E}=V_{b}+I R$; battery voltage $V_{b}=I r, r$ internal battery resistance
b. Junction: Connection of 3 or more conductors
c. Loop: A closed conductor path
d. Resistors in series or parallel $\Rightarrow>$ replace with $R_{\text {tot }}$
e. Capacitors in series or parallel $\Rightarrow>$ replace with $C_{\text {tot }}$
3. Kirchoff's Circuit Rules
a. For any Loop: $\Sigma V=\Sigma I R$;

OHint: Conserve energy b. For any Junction: $\Sigma I=0$;

OHint: Conserve charge;


$$
\boldsymbol{I}_{1}=\boldsymbol{I}_{2}+\boldsymbol{I}_{3}
$$

## G. Magnetic Field: B

1. Key concepts:
a. Moving charge $=>$ Magnetic Field $B$
b. Magnetic Flux: $\Phi_{m}=\oint B d A$
c. Force on charge, $q$ and $v$, moving in $B$ : $F=q v \times B=q v B \sin \theta ; \boldsymbol{v}$ parallel to $\boldsymbol{B}=>=0 ; \boldsymbol{v}$ perpendicular to $\boldsymbol{B}=>F=q v B$
d. Magnetic Moment of a Loop: $M=I A$
e. Torque on a loop: $\tau=\boldsymbol{M} \times \boldsymbol{B}$
f. Magnetic Potential Energy: $U=-\boldsymbol{M} \cdot \boldsymbol{B}$
g. Lorentz Force: Charge interacts with $E$ and $B$;
$\boldsymbol{F}=q \boldsymbol{E}+q \boldsymbol{v} \times \boldsymbol{B}$
H. Faraday's Law and Electromagnetic Induction Key Equations:
2. Faraday's Law: Induced EMF: $\mathrm{E}=\oint E d s=$ $-d \Phi_{m} / d t$
3. Biot-Savart Law: Conductor induces $B$; current $I$, length $d L: d B=\frac{\mu_{0}}{4 \pi} I d L \times \frac{r}{r^{3}}$
4. Sample: Long conducting wire: $B(r)=\frac{\mu_{0}}{4 \pi} \frac{I}{r}$
I. Electromagnetic Waves- Key Equations and Concepts: 1. Transverse $B$ and $E$ fields; $\frac{E}{B}=c$

5. $c=\frac{1}{\sqrt{\mu_{0} \varepsilon_{0}}}$
6. Electromagnetic Wave: $c=f \lambda$ fig 46
J. Maxwell's Equations:
7. Gauss's Law for Electrostatics: $\oint E \cdot d A=\frac{Q}{\varepsilon_{0}}$;
key: Charge gives rise to $\boldsymbol{E}$
8. Gauss's Law for Magnetism: $\oint B \cdot d A=0$;
key: Absence of magnetic charge
9. Ampere-Maxwell Law:
$\oint B \cdot d s=\mu_{0} I+\mu_{0} \varepsilon_{0} \frac{d \Phi_{e}}{d t} ;$
key: Current + change in electric flux $=>B$

## : FHAMOR OF HCHT

A. Basic Properties of Light

1. Goal: Examine light and its interaction with matter
2. Key variables:
a. $c$ : speed of light in a vacuum
b. Index of refraction: $n ; \frac{c}{n}=$ speed of light in medium
c. Light as electromagnetic wave: $\lambda f=c$

Light characterized by "color" or wavelength
d. Light as particle: $e=h f$; energy of photon
3. Reflection and Refraction of Light fig 47
a. Law of Reflection: $\boldsymbol{\theta}_{1}=\boldsymbol{\theta}_{r}$ fig 48
b. Refraction: Bending of light ray as it passes from $n_{1}$ to $n_{2}$
-Snell's Law: $n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}, n_{1}, n_{2}$ are the indices of refraction of two materials fig 49
c. Internal Reflectance: $\sin \theta_{c}=\frac{n_{2}}{n_{1}}$

Light passing from material of higher $\boldsymbol{n}$ to a lower $\boldsymbol{n}$ may be trapped in the material
4. Polarized light: $\boldsymbol{E}$ field is not spherically symmetric
a. Examples: Plane/linear polarized, circularly polarized
b. Polarization by reflection from a dielectric surface at angle $\theta_{c}$; Glass Brewster's Law: $\tan \theta_{c}=\frac{n_{2}}{n_{1}}$

## B. Lenses and Optical Instruments

1. Goal: Lenses and mirrors generate images of objects
2. Key concepts and variables
a. Radius of curvature: $R=2 f$

Lens and Mirror Properties
b. Optic axis: Line from base of object through center of lens or mirror
c. Magnification: $\mathrm{M}=\frac{s^{\prime}}{s}$
d. Laws of Geometric Optics:
$\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} ; \frac{s}{s^{\prime}}=-\frac{h}{h^{\prime}}$
e. Combination of 2 thin lenses:

$$
\frac{1}{f}=\frac{1}{f_{1}}+\frac{1}{f_{2}} \text { or } f=\frac{f_{1} f_{2}}{f_{1}+f_{2}}
$$

| Parameters | + sign | - sign |
| :---: | :---: | :---: |
| $f$ focal length | converging lens <br> concave mirror | diverging lens <br> convex mirror |
| $s$ object dist. | real object | virtual object |
| $s^{\prime}$ image dist. | real image | virtual image |
| $h$ object size | erect | inverted |
| $h^{\prime}$ image size | erect | inverted |

3. Sample Guidelines for ray tracing:
a. Rays that parallel optic axis pass through " $f$ "
b. Rays pass through center of the lens unchanged
c. Image forms at convergence of ray tracings

Q Sample ray tracings: fig 50, a,b,c


## C. Interference of Light Waves

1. Goal: Examine constructive and destructive interference of light waves
2. Key Variables and Concepts:
a. Constructive interference: fig 51
b. Destructive interference: fig 52
c. Huygens' Principle: Each portion of wave front acts as a source of new waves
3. Diffraction of light, from a grating with spacing $d$, produces an interference pattern; $d \sin \theta=\mathrm{m} \lambda ;(\mathrm{m}=0,1,2,3, \ldots)$
4. Single-slit experiment, slit width $a$; destructive interference
 for $\sin \theta=\frac{m \lambda}{a} ;(m=0, \pm 1, \pm 2 \ldots)$

